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VÝKONOVÉ ZTRÁTY V OPTICKÉ ČÁSTI SPOJE PRACUJÍCÍHO VE VOLNÉM PROSTORU

POWER LOSSES IN OPTICAL PART OF THE LINK WORKING IN FREE SPACE

DIPLOMOVÁ PRÁCE
MASTER'S THESIS

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ABSTRAKT

Tato diplomová práce pojednává o charakteristikách laserových diod, difrakcí a o vlastnostech gausovského svazku. Je určen útlum vazby vysílací dioda - vysílací apertura v závislosti na velikostech apertury a na velikosti svazku v programu MATLAB. Jsou stanoveny optimální parametry vazby apertura-dioda.

KLÍČOVÉ SLOVÁ

Laserová dioda, difrakce, gausovský svazek, optické komunikace, výkonové ztráty na apertuře, optimální parametry vazby apertura vysílací dioda

ABSTRACT

This master's thesis deals with the characteristics of the laser diode, diffraction and the properties of the gaussian beam. The coupling attenuation of the laser diode and the transmitting aperture is calculated depending on the sizes of the aperture and on the size of the beam in program MATLAB. The optimal parameters of the coupling diode-aperture are determined.

KEYWORDS

Laser diode, diffraction, gaussian beam, optical communication, power losses on the aperture, optimal parameters of coupling aperture transmitter diode

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Prohlášení

Prohlašuji, že svůj semestrální projekt na téma Výkonové ztráty v optické části spoje pracujícího ve volném prostoru jsem vypracoval samostatně pod vedením vedoucího semestrálního projektu a s použitím odborné literatury a dalších informačních zdrojů, které jsou všechny citovány v práci a uvedeny v seznamu literatury na konci práce.

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V Brně dne 20. května 2011 ....

podpis autora

Poděkování

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1 Intro

Free space optical link is an optical link working in atmosphere. With the advent of the laser diode, free space optical (FSO) links represent a continually more attractive form of communication. Today FSO links are an interesting alternative to other link types, such as radio or wire connection. They have many advantages over other types of communication - allows relatively high data transfer speeds, simple installation and low maintenance cost.

In addition to the attenuations caused by propagation a power losses occur in the communication heads of the link. Such losses are caused by coupling of each elements, or losses on the elements. Calculation of the attenuation diagram of the link requires to know these values.

In this thesis the characteristics and parameters of the laser diode will be studied, the diffraction and the properties of the Gaussian beam will be described. The coupling attenuation of the laser diode and transmitter aperture will be calculated. The optimal parameters of the coupling diode-aperture are determined.

2 Diffraction

Diffraction is a phenomenon where the propagation of the waves bends in a different manner than described with refraction or reflection. The explanation of the diffraction is derived form the Huygens-Fresnel principle. Each point of an advancing wavefront is in fact the center of a fresh disturbance and the source of a new train of waves; additionally, the advancing wave as a whole may be regarded as the sum of all the secondary waves arising from points in the medium already traversed [interntet http://www.ask.com/wiki/Huygens-Fresnel_principle]. The Huygens-Fresnel principle is shown on figure 1.1

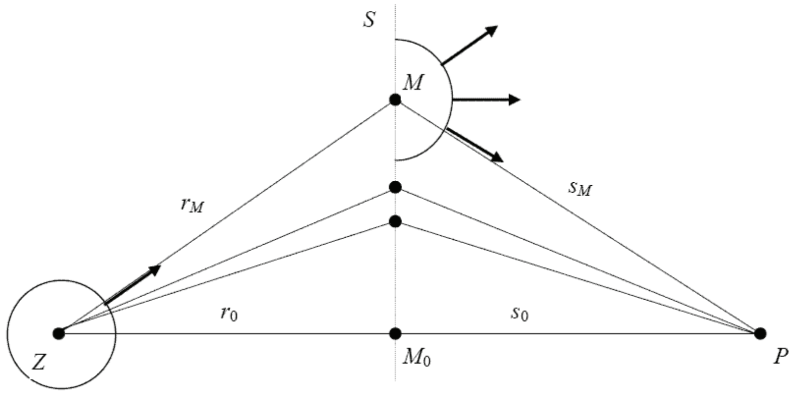


Fig.1.1: the Huygens-Fresnel principle. [1].
Z is the source of the wave, *S* is the surface defining former position of the wave, *P* is the point, in which the resulting waves are formulated, coming from all points of surface *S*, for example from point *M*, r_m, s_m, r_0, s_0 are the lenghts used in mathematical formulation of Huygens-Fresnel principle.

Oscillations of a light wave in each of its points can be considered as the sum of the elementary movements, which are sent at the same time independently form each other by all parts of the considered wave in its earlier position [1].
Huygens-Fresnel principle can be formulated by expression

$$E(P) \approx \int_S K(M) E(M) \frac{e^{jks_M}}{s_M} dS \tag{1.1}$$

where *K(M)* je inclination factor, which gives the contribute loss ratio of the waves according to the distance of *M* and M_0, E(M) is the intensity of electrical field representing primary spherical wave $\frac{e^{jkr_M}}{r_M}$ and k is wave number $\frac{2\pi}{\lambda}$.

At sufficiently small distances $(\lambda \leq K(M) \leq r_0, s_0)$ is $K(M) \approx 1$

By studying diffraction on an aperture in plane shade, the plane *S* of the shade matches with plane $\xi 0 \eta$, and the origin of the coordinate system is in point M_0

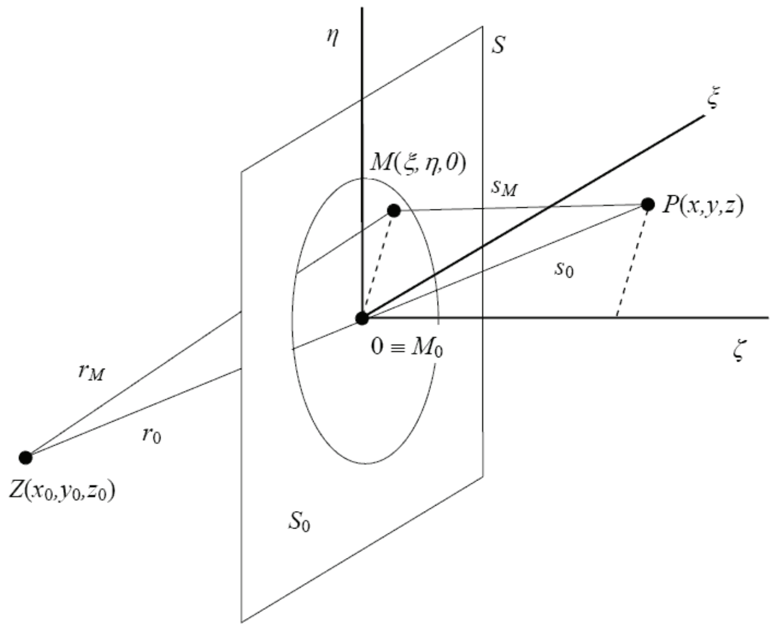


Fig. 1.2: Representation of the diffraction on a circular aperture. [1]

If the Rayleigh criterium is

$$\frac{\left[(x_0-\xi)^2+(y_0-\eta)^2\right]^2}{8z_0^3}\le 0{,}1\lambda \tag{1.2}$$

$$\frac{\left[(x-\xi)^2+(y-\eta)^2\right]^2}{8z_0^3}\le 0{,}1\lambda \tag{1.3}$$

one can write for the Fresnel diffraction:

$$E(P)=-\frac{j}{\lambda}\frac{1}{r_0 s_0}e^{jk(z_0+z)}\int_{S_0} e^{jkf(\xi,\iota)}d\xi d\eta \tag{1.4}$$

where

$$f(\xi,\eta)=\frac{(x_0-\xi)^2+(y_0-\eta)^2}{2z_0}+\frac{(x-\xi)^2+(y-n)^2}{2z} \tag{1.5}$$

and $-j/\lambda$ is the proportion constant

If

$$\frac{\xi^2+\eta^2}{2z_0} \le 0{,}1\lambda \tag{1.6}$$

$$\frac{\xi^2+\eta^2}{2z} \le 0{,}1\lambda \tag{1.7}$$

one can write for the Fraunhofer diffraction:

$$E(P) = C\int_{S_0} e^{-jk\left(\frac{x_0\xi+y_0\eta}{z_0}+\frac{x\xi+y\eta}{z}\right)} d\xi d\eta \tag{1.8}$$

where

$$C = -\frac{j}{\lambda}\frac{1}{r_0 s_0} e^{jk(z_0+z)} e^{jk\left(\frac{x_0^2+y_0^2}{2z_0}+\frac{x^2+y^2}{2z}\right)} \tag{1.9}$$

According to (1.7) condition for the Fraunhofer diffraction is, where *a* is the aperture radius

$$a \le 0{,}2\sqrt{\lambda z} \tag{1.11}$$

which defines the boundary between Fraunhofer and Fresnel diffraction

$$z = \frac{5a^2}{\lambda} = z_0 \tag{1.12}$$

where z_0 is the boundary of near and far field radiation

Criteria for the Fraunhofer diffraction can be formulated by Fresnel number:

$$N_F = \frac{a^2}{\lambda z} \tag{1.13}$$

For $N_F << 0.5$ the Fraunhofer diffraction is obtained. [5] With increasing N_F the diffraction passes to Fresnel diffraction. For large N_F the diffraction pattern is perfect shadow of the aperture. [4] Fig. 1.3 shows the transition from Fresnel diffraction to Fraunhofer diffraction.

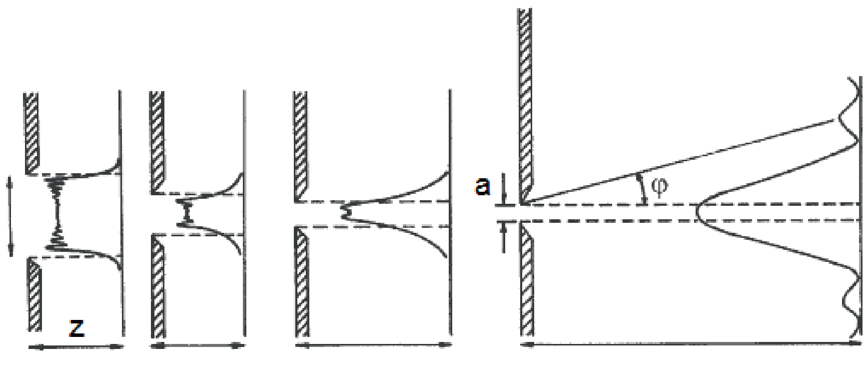


Fig.1.3.: from left to right: transition from Fresnel to Fraunhofer diffraction [8]

3 Characteristics of the laser diode

3.1 Directionality

Very important property of the diode is the high directionality of the emitted light. Instead of LED, where the power distribution is given by angular function *cos φ*, the power distribution of a laser diode can be modelled by function [2]:

$$P_{out} = \cos^{n} \varphi \ [\mathrm{W}] \qquad (2.1)$$

where n is a large number.

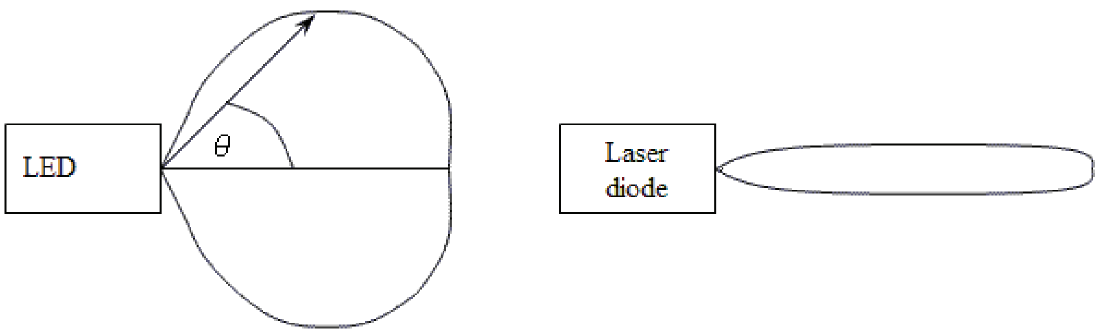


Fig. 2.1.: angular distribution pattern of LED and laser diode

The beam will be highly directional for large n. The reason of the directionality lies in the principle of stimulated emission. The photons are added to the beam with the same phase, thus the entire optical wavefront is oscillating in a synchronized fashion.

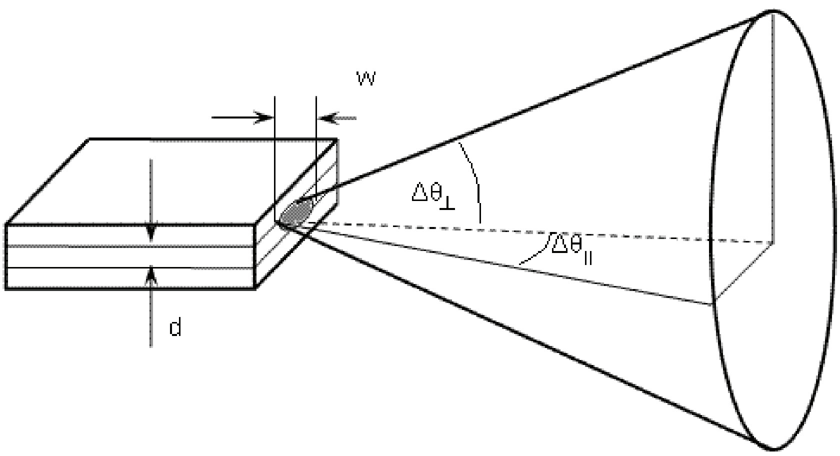


Fig 2.2.: beam dispersion

Beam dispersion in the direction perpendicular to the P-N junction is given by the expression: (fig. 2.2)

$$\Delta\theta_{\perp} \approx \frac{\lambda}{d} \ [°] \tag{2.2}$$

,

where λ is the wavelength and $\Delta\theta$ is the half width of the beam.

Beam dispersion in the direction parallel to the P-N junction is given by the expression (fig. 2.2)

$$\Delta\theta_{\parallel} \approx \frac{\lambda}{w} \ [°] \tag{2.3}$$

where w is the width of the P-N junction.

For the most laser diodes, the width of the d is smaller than w. The angular dispersion is asymmetric, with greater spreading in the direction perpendicular to the P-N layers. The beam has elliptical symmetry. This phenomenon causes complications for coupling the light from the diode, although special aspherical lenses can circularize the beam.

3.2 Spectral characteristics

In comparison with the light of the LEDs, the spectral bandwidth of the light emitted from a laser diode is significantly narrower. Due to the coherence, photons generated at stimulated emission have the same phase and frequency. The spectrum of the light has very narrow bandwidth. The spectrum of the light at high resolutions has a mode structure. The mode structure is shown on fig. 2.3.

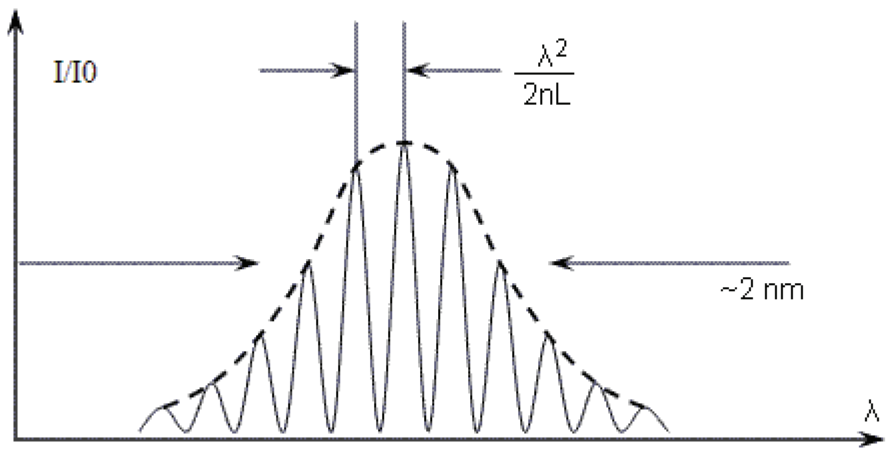


Fig. 2.3.: light spectrum of a laser diode

Mode spacing is given by expression [2]

$$c/(2nL) \tag{2.4}$$

where *n* is the refraction index
and *L* is the resonator index.

Corresponding mode spacing in wavelengths is given by expression is [2]

$$\lambda/(2nL) \tag{2.5}$$

where λ is the wavelength. At multimode operation the width of the spectrum is given by the sum of each modes. The narrow spectral bandwidth of the laser diode is a great advantage in optical communications, especially when using wavelength division multiplexing (WDM).

4 Gaussian beam

For laser links working in free space is important that the optical wavefronts have to spread in a narrow beam, for example along the z axis in an xyz system of coordinates. Waves, whose normals contain a small angle with the z axis are paraxial waves. Such waves have to satisfy the wave equation. An important solution for the paraxial waves is the gaussian beam. The optical power is concentrated into a narrow beam, and the optical intensity in the plane perpendicular to the direction of the wave propagation is given by circularly symmetrical Gauss function with maximum I_0 on the axis of the beam (on the z axis) .

Paraxial wave is a wave e^{-jkz} modulated by complex envelope $A(r)$. The complex envelope is given by expression

$$U(r) = A(r) \times \exp(-jkz) \tag{3.1}$$

The envelope is considered approximately constant within the boundary of λ, the wave in certain boundaries can be considered as a plane wave with wavefronts, that are paraxial waves. A plain solution for the paraxial Helmholtz equation is a paraboloidal wave:

$$A(r) = \frac{A_1}{z} \exp\left(-jk\frac{\rho^2}{2z}\right), \rho^2 = x^2 + y^2 \tag{3.2}$$

where A_1 is constant. Another solution for the paraxial Helmholtz equation gives the Gaussian beam. It is derived from the paraboloidal wave with a simple transformation $z \to \xi$.

$$A(r) = \frac{A_1}{q(z)} \exp\left[-jk\frac{\rho^2}{2q(z)}\right], q(z) = z - \xi \tag{3.3}$$

Where z_0 is the Rayleigh distance.

After different transformations of (3.3) the complex amplitude of the Gaussian beam can be expressed:

$$U(r) = A_0 \frac{W_0}{W(z)} \exp\left[-\frac{\rho^2}{W^2(z)}\right] \exp\left[-jkz - jk\frac{\rho^2}{2R(z)} + j\zeta(z)\right] \quad (3.4$$

4.1 Half-width of the beam, angle of divergence

. Relation of the half-width of the beam to the z axis is expressed by relation

$$W(z) = W_0 \left[1 + \left(\frac{z}{z_0} \right)^2 \right]^{0,5} \text{ [m]} \tag{3.5}$$

Its minimum W_0 or the beam waist is in $z = 0$. Value $2W_0$ expresses the width of the beam in the waist. The half width of the beam grows gradually with z, with value $\sqrt{2}W_0$ for z = z_0.

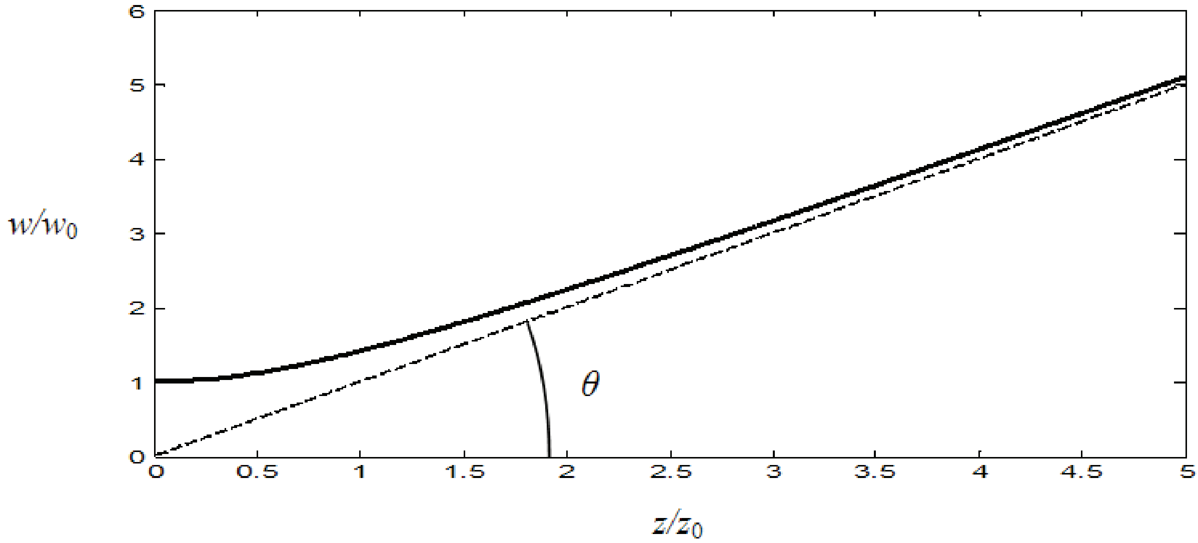


Fig. 3.1.: Relation of the half-width of the beam to the z axis.

Angle θ, contained by the asymptote of the hyperbola with the z axis is called the angle of divergence. It is defined by formula

$$\theta_0 = \frac{2}{\pi} \times \frac{\lambda}{2W_0} \text{ [°]} \tag{3.6}$$

The angle of divergence is directly proportional to the ratio of the wavelength and half width of the beam. Highly directional beam can be achieved by wide waist or short wavelength. Distance, where the half width of the beam is $\sqrt{2}W_0$ is the boundary of near and far field, and is given by relation

$$2z_0 = \frac{2\pi W_0^2}{\lambda} \text{ [m]} \tag{3.7}$$

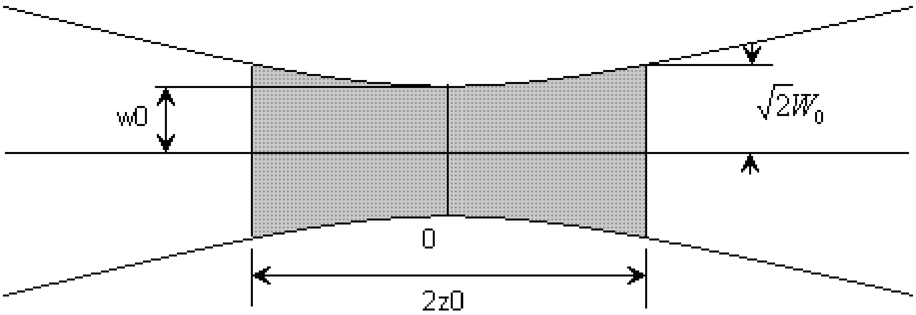


Fig. 3.2.: boundary of near and far field.

For phase shift of the wave *φ(z)*in relation with the axis *z* we can write

$$tg\varphi(z)=\frac{z_0}{z} \tag{3.8}$$

Value of *φ(z)* is $-\pi/2$ for $z = -\infty$ and $\pi/2$ for $z = \infty$. This shift corresponds to the delay of the wavefront compared to the wavefronts os plane or spherical waves. The overall phase shift from $z = -\infty$ do $z = \infty$ is π.

4.2 Wavefronts

Wavefront radius of curvature can be expressed by relation

$$R(z) = z\left[1+\left(\frac{z_0}{z}\right)^2\right] \text{ [m]} \tag{3.9}$$

Wavefront radius of curvature is infinite in $z = 0$, in this case the gaussian beam takes form of a plane wave. It's value decreases to $2z_0$ in the Rayleigh range. At this point have the wavefronts the highest curvature. For $z > z_0$ the radius of curvature increases agan, and for $z >> z_0$ the wavefronts of the beam resemble wavefronts of a spherical wave. Relation of $R(z)$ to z shows figure 3.3

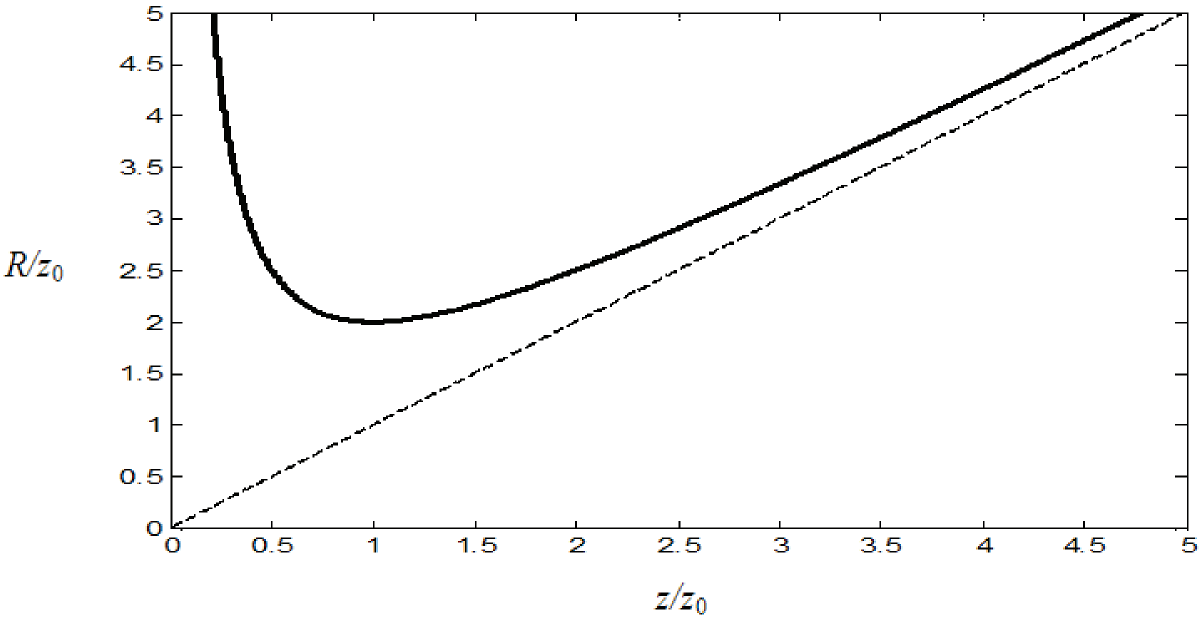


Fig. 3.3.:Relation of wavefront radius $R(z)$ to non-dimensional coordinates z/z_0

4.3 Intensity

The optical intensity $I(r) = \left|U(r)^2\right|$ is a function of distance z and $\rho = \left(x^2 + y^2\right)^{1/2}$

$$I(\rho, z) = I_0 \left[\frac{W_0}{W(z)}\right]^2 \exp\left[-\frac{2\rho^2}{W^2(z)}\right] [\mathrm{W/m^2}] \tag{3.10}$$

where I_0 is the optical intensity on the beam axis in the origin ($z = 0$). For each value of z the intensity distribution is gaussian. The gaussian distribution has it's peak on axis ($\rho = 0$) and monotonically decreases with the distance ρ. The width of the gaussian distribution grows with distance z, see. Fig 3.1.

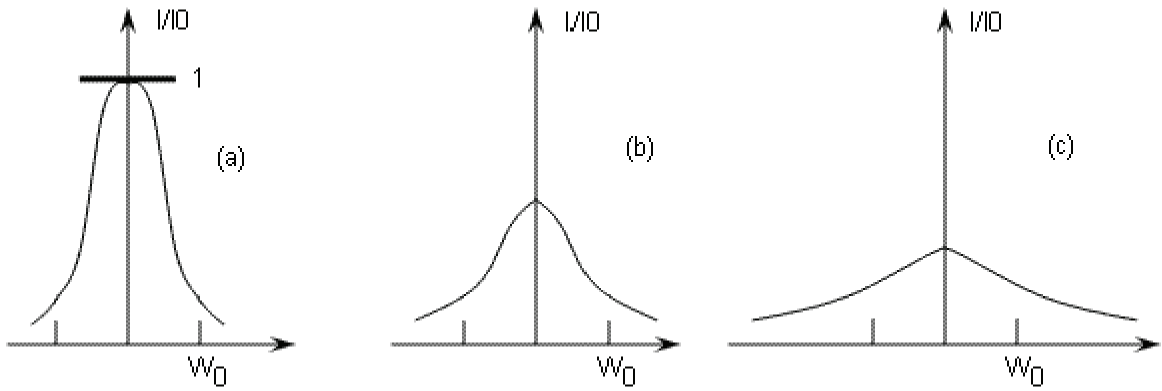


Fig.3.4.: Beam intensity for different distances of z: (a) $z = 0$, (b) $z = z_0$, (c) $z = 2z_0$

On the beam axis the optical intensity is given by relation

$$I(0, z) = I_0 \left[\frac{W_0}{W(z)}\right]^2 = \frac{I_0}{1 + (z / z_0)^2} \tag{3.11}$$

The beam intensity reaches the maximum value on the beam axis and decreases according to the factor $1/e^2 \approx 0{,}135$ at distance $\rho = W(z)$, which is the half width of the beam. Half value of I_0 the optical intensity reaches in at distance $\pm z_0$. In case of $|z| >> z_0$ the intensity is:

$$I(0, z) \approx I_0 z_0^2 \times \frac{1}{z^2} \tag{3.12}$$

which means, that the optical intensity decreases with the quadrant of the distance as in the case of spherical or paraboloidal waves.

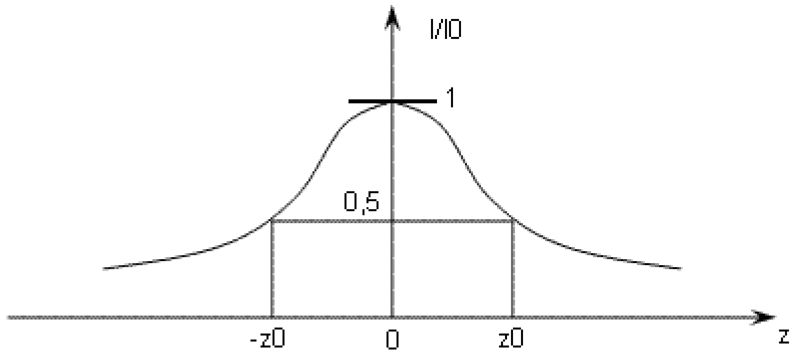


Fig. 3.5.: Intensity I/I_0 as function of coordinate *z.*

4.4 Power

The total optical power carried in the beam is an integral of optical intensity over transverse plane

$$P = \int_0^\infty I(\rho, z) 2\pi\rho d\rho \text{ [W]} \tag{3.13}$$

,

After evaluating the integral the optical power can be expressed

$$P = \frac{1}{2} I_0 \pi W_0^2 \tag{3.14}$$

The result is independent from the distance *z.* Approximately 86% of the power is carried in the beam with half width of *W(z)*, and and approximately 99% of the power is carried in the beam with half width of 1,5*W(z)* [3]

The power contained in the beam of the laser diode where the beam has elliptical syymetry can be expressed as [9]:

$$P = \int_S I(\rho) ds = \iint_{R^2} I(x, y) dxdy = \iint_{R^2} I_0 e^{-2\left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right)} dxdy \text{ [W]} \tag{3.15}$$

where $I_0 e^{-2\left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right)}$ is the intensity distribution function of a laser beam with elliptical symmetry and intensity with Gaussian distribution.

5 Modelling the power losses in optical part of the link working in free space

Free Space Optics (FSO) is an optical communication technology that uses light propagating in free space to transmit data between two points.[wikipedia]. FSO links are based on two RX/TX heads communicating in full duplex mode. The optical power is concentrated into one or multiple narrow beams containing one or multiple wave-separated channels [7]

.When designing a link, the data transfer rate, bit error rate, link distance, attenuations and the parameters of the devices have to be considered. FSO links have their specific problems. In the case of terrestrial optical links there are various phenomena causing attenuations, for example atmospheric absorption, turbulence, snow, rain or fog. The appropriate communication devices have to be chosen. Apart from the attenuations caused by the propagation, other power losses have to be considered, such as attenuations in the communication heads.

An example of a communication head is shown on figure 4.1.:

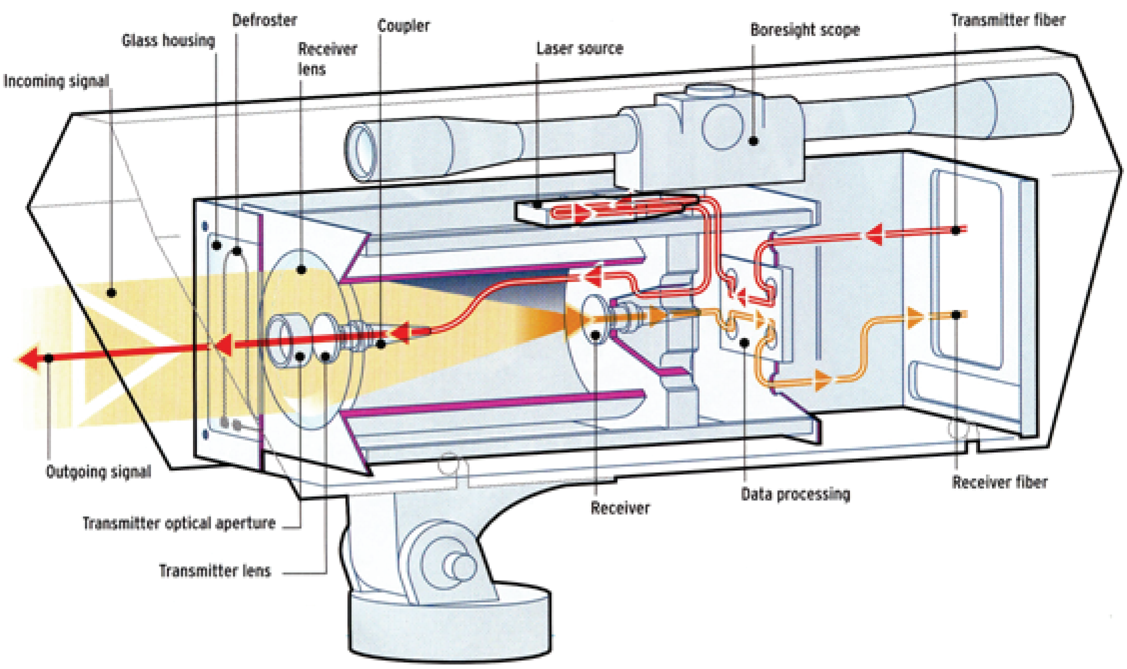


Fig. 4.1.: structure of a FSO communication head. [7]

5.1 Truncated and obscured beam

The power contained in the untruncated Gaussian beam can be expressed as the integration of the intensity distribution function over an area defined by the size of the beam cross section in the direction of the *z* axis :

$$P = \int_S I(\rho) dS \tag{4.1}$$

where $I(\rho)$ is the intensity distribution function of a laser beam with circular symmetry and intensity with Gaussian distribution, I_0 is the intensity in point $x,y=0$ and R^2 is the area of beam cross section and *w* is the half-width of the beam. After evaluating the integral, one can arrive to the fact, that total power is carried in the beam when the integral is evaluated on the whole plane. Practically it means that in a free space optics link total power transmission occurs only with an aperture with infinite size.

However, real systems do not have infinite apertures. In addition to truncation, an optical train may be used which has beam obscuring components [6] causing additional power loss. Such obscuring component may be the transmitter aperture in the center of the receiver aperture in the FSO communication head (fig. 4.1.). The problem is determining optimum lens diameter considering all power losses.

Figure 4.2. shows the interaction of the aperture with the beam. The aperture is considered as a circularly symmetric hole with homogenous transmission in each point.

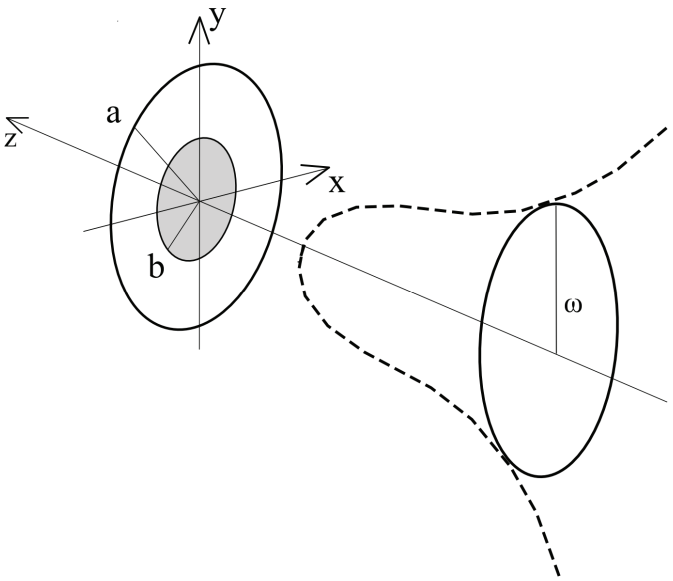


Figure 4.2.: interaction of the aperture with the Gaussian beam.
a is the aperture radius, *b* is the radius of the obscuring component, *ω* is the half width of the beam. The dashed line indicates the envelope of the Gaussian beam.

Expression (3.15) should be used for the description of the interaction aperture - Gaussian beam, if the source of the beam is a laser diode. However, this would lead to complex numerical evaluations. Considering a simplified case - circulalry symmetric beam, truncation and obscuring component, the output power from the aperture can be written [6]:

$$P_{out} = 2\pi \int_{\varepsilon a}^{a} u_0^2 e^{\frac{-2\rho^2}{\omega}} \rho d\rho \Rightarrow P_{out} = 2A_T u_0^2 \int_{\varepsilon}^{1} e^{-2\xi_r{}^2 x^2} x dx \tag{4.2}$$

where a is the aperture size, ω is the beam's cross section radius or the half-width of the beam, b is the radius of the obscuring component , $\varepsilon = b/a$ is the ratio of the inner and outer radii of the aperture, $\xi_r = a/\omega$ is the aperture-to-beam radius ratio, u_0 is the amplitude of the beam, A_T is the area of the transmitting aperture.

If $u_0^2 = I_0 = 2P_{in} \big/ \pi\omega^2$, it is possible to write [6]:

$$\frac{P_{out}}{P_{in}} = e^{-\varepsilon^2 \xi_r{}^2} - e^{-2\xi_r{}^2} \tag{4.3}$$

where P_{out} is the output power, and P_{in} is the input power impinging on the aperture.

Expression (4.3) allows to calculate power transmission depending on the aperture size, beam half width or the size of the obscuring component.

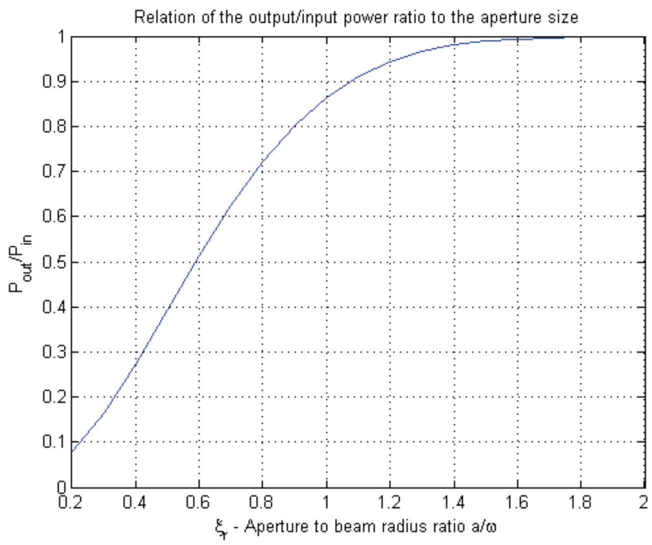
5.2 Calculating the power transmission

For calculating the output and input power ratio the MATLAB software was used. It is very effective tool for evaluating the values of given functions, furthermore it is possible to represent the values of the functions on graphs. Setting the range of variables, performing calculations or creating graphs are done by simple commands. According to (4.3) the power output ratio depends on more parameters. In the further section the effects of these parameters will be examined.

In the first case when there is no obscuring component, expression (4.3)simplifies into:

$$\frac{p_{out}}{P_{in}} = 1 - e^{-2\xi_r^2} \qquad (4.4)$$

If the beam size is constant, the output/input power ratio depends on the size of the aperture. Figure 4.3., table 4.1. shows the computed values for different ξ_r.
Half-width of the beam is set to $\omega = 50$ [-]



a	ξ_r	*Pout/Pin*
10,000	0,200	0,077
15,000	0,300	0,165
20,000	0,400	0,274
25,000	0,500	0,393
30,000	0,600	0,513
35,000	0,700	0,625
40,000	0,800	0,722
45,000	0,900	0,802
50,000	1,000	0,865
55,000	1,100	0,911
60,000	1,200	0,944
65,000	1,300	0,966
70,000	1,400	0,980
75,000	1,500	0,989
80,000	1,600	0,994
85,000	1,700	0,997
90,000	1,800	0,998
95,000	1,900	0,999
100,000	2,000	1,000

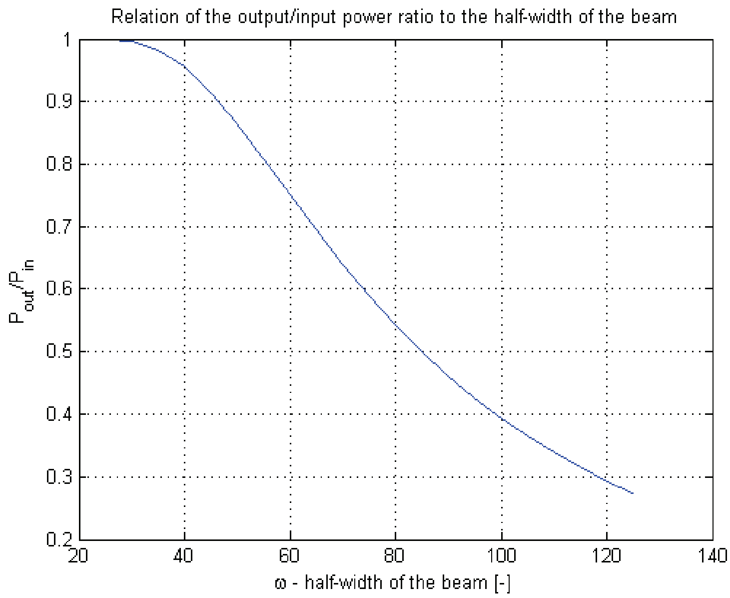
Fig. 4.3., tab. 4.1.: input/output power ratio for different ξ. (appendix 1.a)

As expected, maximum power transmission occurs with aperture with infinite radius. When $\xi_r = 1{,}5$, approximately 99% of the power is transmitted, when $\xi_r = 1$, approximately 87% of the power is transmitted, which is evident according to chapter 3.4. Ratio of the input and output power falls with the increasing truncation.

The power transmission ratio can be calculated form a different approach, when the radius of the aperture is fixed. It is obvious, that the power loss is caused by the truncation again.

Figure 4.4., table 4.2. shows the input/output power ratio for different beam size.

Radius of the aperture is set to $a = 50$ [-].



ω	ξ_r	*Pout/Pin*
25,000	2,000	1,000
30,000	1,667	0,996
35,000	1,429	0,983
40,000	1,250	0,956
45,000	1,111	0,915
50,000	1,000	0,865
55,000	0,909	0,809
60,000	0,833	0,751
65,000	0,769	0,694
70,000	0,714	0,640
75,000	0,667	0,589
80,000	0,625	0,542
85,000	0,588	0,499
90,000	0,556	0,461
95,000	0,526	0,425
100,000	0,500	0,393
105,000	0,476	0,365
110,000	0,455	0,338
115,000	0,435	0,315
120,000	0,417	0,293
125,000	0,400	0,274

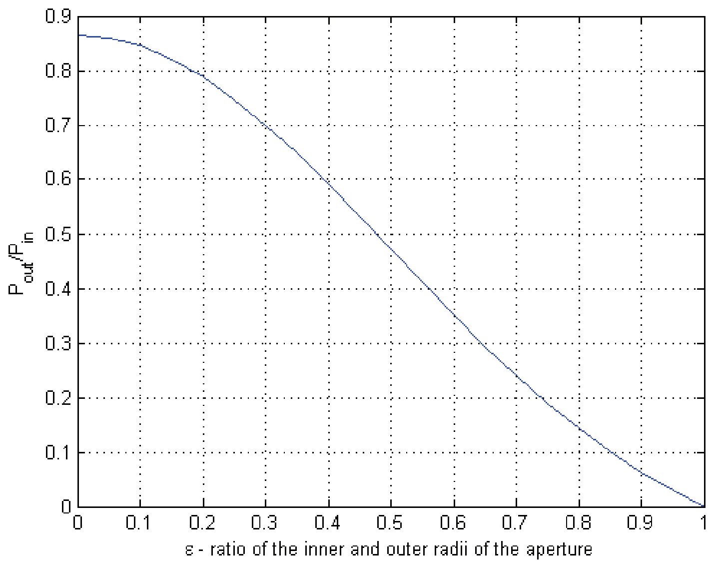
Fig. 4.4., tab. 4.2.: : input/output power ratio for different ω (appendix 1.b)

Different situation occurs in a presence of an obscuring component. If the size of the beam and aperture is fixed, the power transmission is determined according to the size of the obscuring component.

Figure 4.5, table 4.3 shows shows the input/output power ratio for different radius of the obscuring component.

Aperture size is set to $a = 100$

half-width of the beam is set to $\omega = 100$

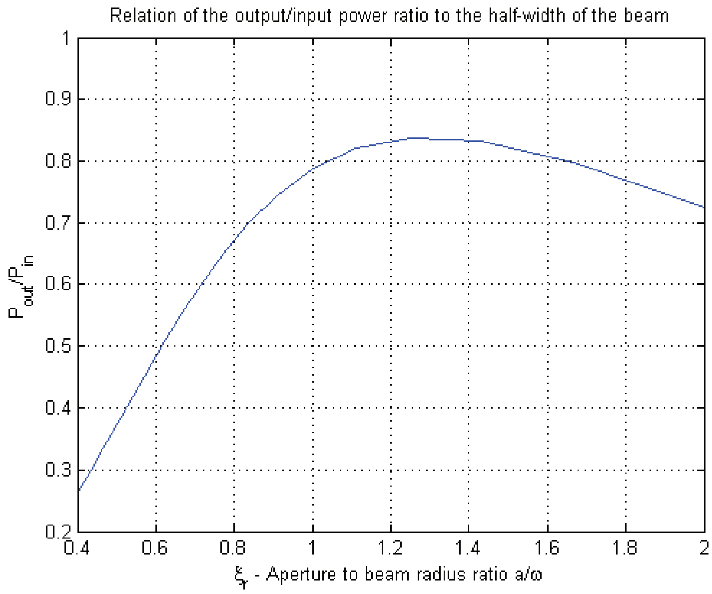


b	*ε*	*Pout/Pin*
0,000	0,000	0,865
5,000	0,050	0,860
10,000	0,100	0,845
15,000	0,150	0,821
20,000	0,200	0,788
25,000	0,250	0,747
30,000	0,300	0,700
35,000	0,350	0,647
40,000	0,400	0,591
45,000	0,450	0,532
50,000	0,500	0,471
55,000	0,550	0,411
60,000	0,600	0,351
65,000	0,650	0,294
70,000	0,700	0,240
75,000	0,750	0,189
80,000	0,800	0,143
85,000	0,850	0,100
90,000	0,900	0,063
95,000	0,950	0,029
100,000	1,000	0,000

Fig. 4.5., tab. 4.3.: input/output power ratio for different *ε* (appendix 1.c)

In this case, the increasing size of the obscuring component decreases the transmitted power.

One may want to calculate the power transmission depending on the size of the beam when the obscuring component is present. Equation (4.3) is used for the calculation. Surprisingly the ratio of the output and input power raises only to a certain point. Beyond this point the amount of transmitted power begins to decrease.



ω	ξ_r	Pout/Pin
25,000	2,000	0,726
30,000	1,667	0,797
35,000	1,429	0,832
40,000	1,250	0,839
45,000	1,111	0,821
50,000	1,000	0,788
55,000	0,909	0,745
60,000	0,833	0,697
65,000	0,769	0,648
70,000	0,714	0,600
75,000	0,667	0,554
80,000	0,625	0,511
85,000	0,588	0,472
90,000	0,556	0,436
95,000	0,526	0,403
100,000	0,500	0,374
105,000	0,476	0,347
110,000	0,455	0,322
115,000	0,435	0,300
120,000	0,417	0,280
125,000	0,400	0,261

Fig. 4.6., tab. 4.4.: input/output power ratio for different ω, $\varepsilon = 0.2$ (appendix 1.d)

Figure 4.6 shows the power transmission of the aperture, where te size of the obscuring component is b =10.

5.3 Determining the optimal aperture diameter.

Figure 4.7. shows the relation of the output/input power to the aperture to beam radius ratio for different sizes of the obscuring component. Aperture radius is *a=50* and the half-width of the beam ω is in range 25-125. Maximum power transmission occurs at different values of ξ_r for each ε. This fact leads to a theory, that there must be an optimum aperture to beam ratio giving maximum power transmission for a certain size of an obscuring component.

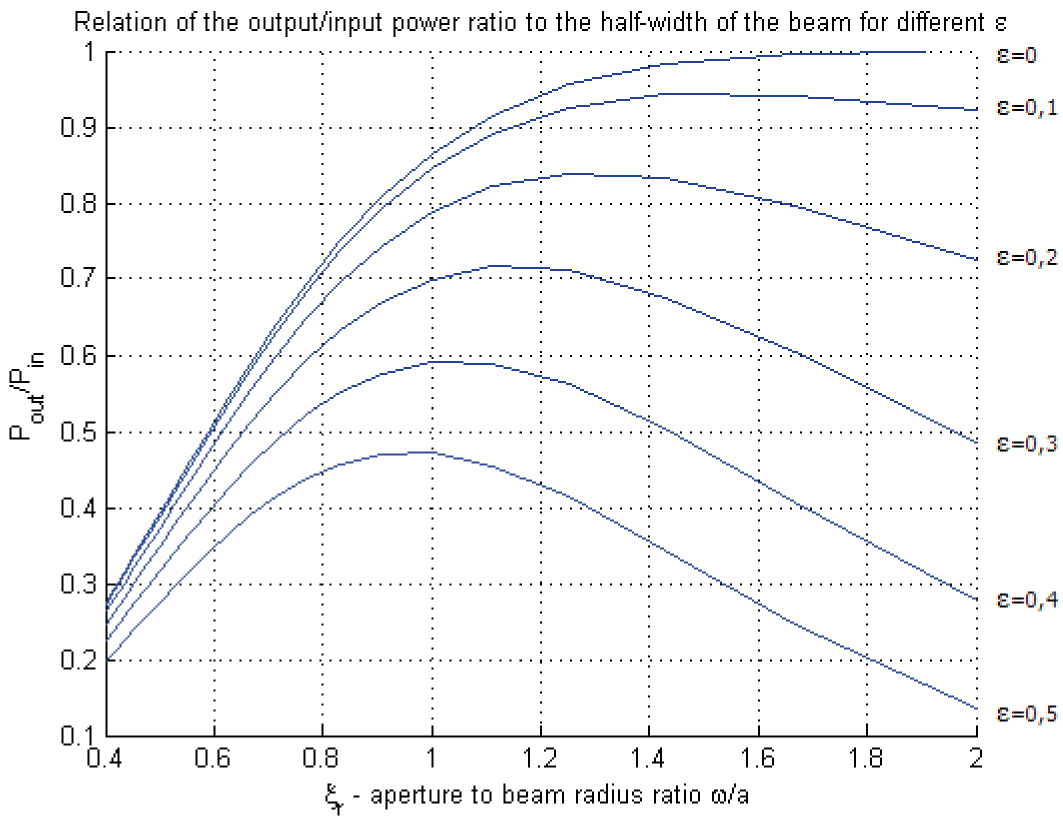


Figure 4.7.: relation of the ratio of the output/input power to ξ_r for different ε. (appendix 1.e)

At extreme cases, for example when $\varepsilon = 0{,}6$, the optimal aperture to beam radius ratio $\xi_r < 1$, which means that for maximum power transmission the aperture radius must be smaller, than the half width of the beam.

		Pout/Pin					
ω	ξ_r	b=0	b=5	b=10	b=15	b=20	b=25
25,000	2,000	1,000	0,923	0,726	0,486	0,278	0,135
30,000	1,667	0,996	0,942	0,797	0,603	0,407	0,245
35,000	1,429	0,983	0,943	0,832	0,676	0,504	0,344
40,000	1,250	0,956	0,925	0,839	0,711	0,563	0,414
45,000	1,111	0,915	0,891	0,821	0,716	0,589	0,455
50,000	1,000	0,865	0,845	0,788	0,700	0,591	0,471
55,000	0,909	0,809	0,792	0,745	0,670	0,576	0,470
60,000	0,833	0,751	0,737	0,697	0,633	0,551	0,457
65,000	0,769	0,694	0,682	0,648	0,593	0,521	0,438
70,000	0,714	0,640	0,629	0,600	0,552	0,489	0,414
75,000	0,667	0,589	0,580	0,554	0,512	0,456	0,390
80,000	0,625	0,542	0,534	0,511	0,474	0,425	0,365
85,000	0,588	0,499	0,493	0,472	0,439	0,395	0,341
90,000	0,556	0,461	0,454	0,436	0,407	0,367	0,318
95,000	0,526	0,425	0,420	0,403	0,377	0,341	0,296
100,000	0,500	0,393	0,388	0,374	0,349	0,317	0,276
105,000	0,476	0,365	0,360	0,347	0,325	0,295	0,257
110,000	0,455	0,338	0,334	0,322	0,302	0,275	0,240
115,000	0,435	0,315	0,300	0,281	0,256	0,225	0,225
120,000	0,417	0,293	0,280	0,263	0,239	0,210	0,210
125,000	0,400	0,274	0,261	0,245	0,224	0,197	0,197

Tab. 4.5.: relation of the ratio of the output/input power to ξ_r for different ε (appendix 1.e)

Table 4.5 shows the calculated values of the power transmission. Values marked with yellow color represent the maximum power transmission for given ε.

In certain situations, the aperture to beam radius ratio must be smaller than in case of an unobscured aperture. This fact seems to be in contrary with the statement that maximum power transmission occurs when the beam size is smaller than the size of the aperture. However, this is in relation with the properties of the beam. According to chapter 3.3, the intensity is a Gaussian function of the radial distance ρ and the intensity has its maximum value at the beam center $\rho = 0$. In other words, the major part of the power contained in the beam is concentrated at the beam center. An obscuring component with size comparable to the beam would block significant amount of the beam power. With increasing beam width, the power spreads into a larger area, hence, less power is blocked by the obscuring component in case of broad beam.

It is possible to compute the optimal aperture diameter in relation of the size of the obscuring component. Taking the derivative of equation (4.3) and setting it equal to zero shows that a maximum occurs at ξ_{ropt}, given by [6]:

$$\xi_{ropt} = \left(\frac{\ln \varepsilon}{\varepsilon^2 - 1} \right)^{\frac{1}{2}} \qquad (4.5)$$

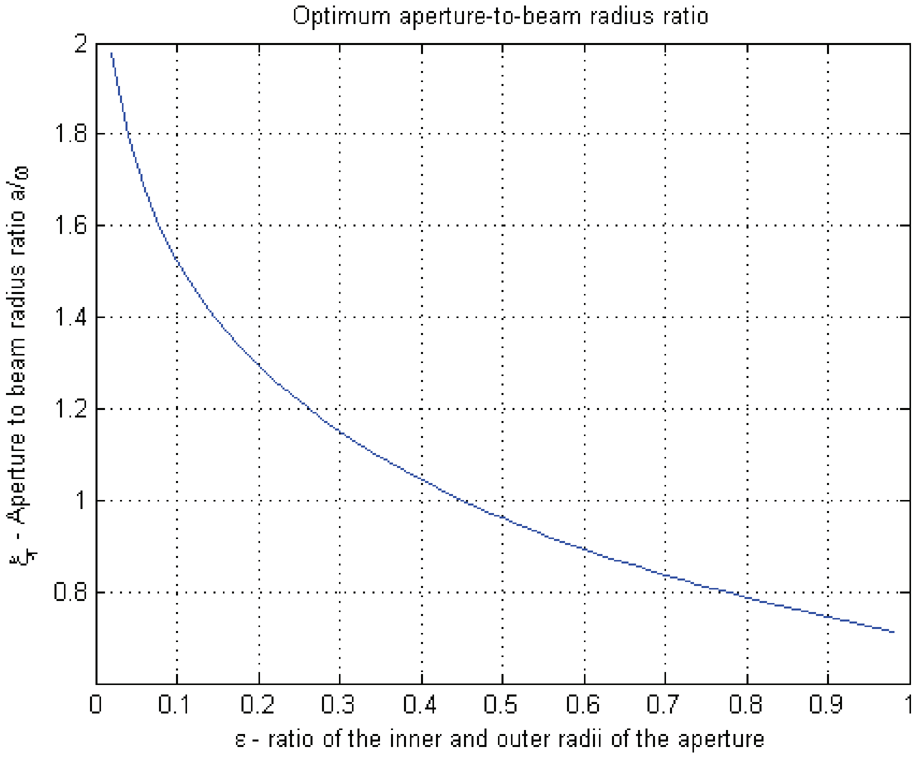


Fig.4.8.: relation of optimum aperture-to-beam radius ratio to the ratio of inner to outer radii of the aperture (appendix 1.f)

Figure 4.6. shows relation of optimum aperture-to-beam radius ratio to the ratio of inner to outer radii of the aperture.

5.3.1 Determining the optimal distance

According to (3.10), the width of the Gaussian distribution increases with the axial distance *z*. The power contained in the beam is the integration of the intensity distribution over the area of the beam and the intensity on the beam axis is a function of radial and axial distance ρ and z. One may eventually arrive to the fact that ξ_{ropt} is in relation with the axial distance of the beam. This means, that there is an optimal distance between the aperture and the source of the beam. Practically, it can be the distance between the transmitter laser diode and aperture.

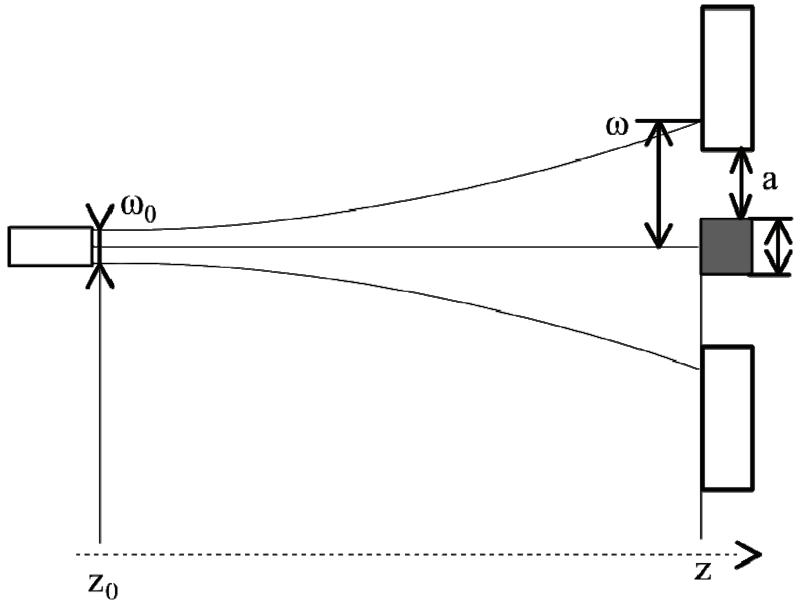


Figure 4.9.: representation of the beam source (left) and aperture (right). The dark gray area is the obscuring component, light grey area defines the boundaries of the aperture, dashed line indicates the *z* axis

Based on the findings gained by previous calculations, the following statements can be said:

According to (3.5):

$W(z) = \omega$, $W(0) = \omega_0$

$$\frac{\omega}{\omega_0} = \left[1 + \left(\frac{z}{z_0}\right)^2\right]^{0,5} \tag{4.6}$$

If a is constant,

$$\xi_r = \frac{a}{\omega} \Rightarrow \xi_{ropt} = \frac{a}{\omega_{opt}} \qquad (4.7)$$

$$\begin{gathered} \left(\frac{\omega_{opt}}{\omega_0}\right) = 1 + \left(\frac{z_{opt}}{z_0}\right)^2 \\ \sqrt{\left(\frac{a}{\xi_{ropt}\omega_0}\right)^2 - 1} = \frac{z_{opt}}{z_0} \end{gathered} \qquad (4.8)$$

where ω_{opt} is the optimal half-width of the beam and $\frac{z_{opt}}{z_0}$ is the optimal relative distance beam source-aperture.

Remark:

When the obscuring component is the same size as the aperture , $\varepsilon \to 1 \Rightarrow \xi r_{opt} \to 0$, according to (4.5), $\omega_{opt} \to \infty$, and $z_{opt} \to \infty$ (4.6),(4.8).

6 Conclusion

This master's thesis dealt with the power losses occuring in the optical part of a laser link working in the atmosphere.

Chapter 2-4 deals with the diffraction, properties of the laser diode and the Gaussian beam. Based on the acquired informations, the characteristics of the laser beam. the coupling attenuation of the laser diode and transmitter aperture was calculated. The aperture was considered circulary symmetric with a circularly symmetric obscuring component in it. The beam impinging on the aperture was a circularly symmetric beam with intensity with Gaussian distribution.

Without the obscuring component the transmitted power through the aperture increases with the increasing size of the aperture, in the other words, power loss increases with increasing half-width of the beam.

Chapter 5 deals with the optimal arrangement of the aperture and beam source. When an obscuring component is in the aperture, the situation is not obvius. Maximum power transmission occurs for a certain half-width of the beam. The optimum aperture-to-beam ratio can be calculated for each size of the obscuring component. Based on the optimum aperture-to-beam ratio the optimum relative distance of beam source-aperture can be determined. Due to lack of time, measurements were not performed. For the best accuracy, the elliptical symmetry of the beam should be considered, also it would be essential to verify calculated values with measurements.

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8 List of acronyms, symbols, appendices

FSO	free space optical
$E(P)$	residual wave in point P
$K(M)$	inclination factor
E(M)	intensity of electric field representing primary spherical wave
r_m, s_m, r_0, s_0	coordinates
$\xi,0,\eta$	coordinates
x,y,z	coordinates
N_F	Fresnel number
a	circural aperture radius
$\Delta\theta_{\perp}$	Beam dispersion in the direction perpendicular to the P-N junction
$\Delta\theta_{\parallel}$	Beam dispersion in the direction parallel to the P-N junction
$U(r)$	complexn amplitude of the paraxial wave
$A(r)$	complex envelope of the paraxial wave
W_0	half-width of the beam
θ	angle of divergence
$2z_0$	boundary of near and far field
$\varphi(z)$	phase shift of the wavefronts
$R(z)$	*w*avefront radius of curvature
$I(z)$	intensity of the Gaussian beam
P	power of the Gaussian beam
RX	receiver
TX	transmitter
a	radius of the aperture
b	radius of the obscuring component
ω	beam half width
ε	the ratio of inner to outer radii of the aperture
ξ_r	the aperture-to-beam radius ratio
ξ_{ropt}	optimum aperture-to-beam radius ratio
ω_{opt}	optimal half-width of the beam
αvdvc	coupling attenuation laser diode/TA
z_{opt}/z_0	optimal relative distance beam source-aperture

9 Appendix: MATLAB source code

a) **Script for calculating the ratio of input and output power without obscuration:**

```
close all
clear all
clc


P0 = 1;                     % ... input power
w = 50;                    % ... beam cross section radius
av = 10:5:100;              % ... radius of the circular aperture
Fv = (av./w);               % ... aperture-to-beam radius ratio


ratio = zeros(length(av));

ratio = P0*(1-exp(-2*Fv.^2));

figure(1)
plot(Fv,ratio); grid on; title ('');...
xlabel ('\xi_r - Aperture to beam radius ratio a/\omega');ylabel...
('P_{out}/P_{in}');
```

b) **Script for calculating the ratio of input and output power according to the beam size:**

```
close all
clear all
clc

P0 = 1;                     % ... power input
a = 50;                     % ... radius of the circular aperture
wv = 25:5:125;              % ... beam cross section radius
Fv = (a./wv);               % ... aperture-to-beam radius ratio


ratio = zeros(length(wv));

ratio = P0*(1-exp(-2*Fv.^2));

figure(1)
plot(wv,ratio); grid on; title ('');...
xlabel ('\xi_r - Aperture to beam radius ratio a/\omega');ylabel...
('P_{out}/P_{in}');
```

c) Script for calculating the ratio of input and output power according to the size of the obscuring component:

```
close all
clear all
clc

P0 = 1;                     % ... power input
b = 10;                     % ... obscuration component size
a = 50;                     % ... radius of the circular aperture
wv = 25:5:125;              % ... beam cross section radius
Fv = (a./wv);               % ... aperture-to-beam radius ratio
gv = b./a;                  % ... ratio of inner to outer radii of the aperture

ratio = zeros(length(wv));

ratio = P0*((exp(-2*gv.^2.*Fv.^2))-exp(-2*Fv.^2));

figure(1)
plot(Fv,ratio); grid on; title ('');...
xlabel ('\xi_r - Aperture to beam radius ratio a/\omega');ylabel...
('P_{out}/P_{in}');
```

d) Script for calculating the the ratio of input and output power according to the size of the beam with obscuration

```
close all
clear all
clc

P0 = 1;                     % ... power input
b = 10;                     % ... obscuration component size
a = 50;                     % ... radius of the circural aperture
wv = 25:5:125;              % ... beam cross section radius
Fv = (a./wv);               % ... aperture-to-beam radius ratio
gv = b./a;                  % ... ratio of inner to outer radii of the aperture

ratio = zeros(length(wv));

ratio = P0*((exp(-2*gv.^2.*Fv.^2))-exp(-2*Fv.^2));

figure(1)
plot(Fv,ratio); grid on; title ('Relation of the output/input power ratio
to the half-width of the beam');...
xlabel ('\xi_r - Aperture to beam radius ratio a/\omega');ylabel...
('P_{out}/P_{in}');
```

***e)* Script for calculating the ratio of input and output power according to the beam size and size of the obscuring component:**

```
close all
clear all
clc

P0 = 1;                    % ... input power
b = 0:5:25;               % ... obscuration component size
wv = 25:5:125;                    % ... beam cross section radius
a = 50;                  % ... radius of the circuar aperture
Fv = (a./wv);              % ... aperture-to-beam radius ratio
gv = b./a;                % ... ratio of inner to outer radii of the aperture
bv = wv./a;

ratio=zeros(length(wv),length(gv));

for i=1:size(ratio,1);
    for j=1:size(ratio,2);
        ratio(i,j)=P0.*((exp(-2.*(gv(j).^2)*(Fv(i).^2)))-exp(-
2.*(Fv(i).^2)));
    end
end




figure(2);

hold on;
for j=1:size(ratio,2)
   plot(Fv,ratio(:,j)); grid on; title ('');...
       xlabel ('\xi_r - aperture to beam radius ratio \omega/a');ylabel ...
     ('P_{out}/P_{in}')
end
hold off;
```

f) Script for calculating the optimum aperture-to-beam radius ratio

```
close all
clear all
clc

a=50;                  % ... aperture size
bv=0:1:49;             % ... obscuring component size


e = bv./a;             %... ratio of inner to outer radii of the aperture

q = ((log(e))./(e.^2-1)).^0.5; % ... Optimum aperture-to-beam radius ratio



figure(1)
plot(e,q); grid on; title ('Optimum aperture-to-beam radius ratio');...
       xlabel ('\epsilon - ratio of the inner and outer radii of the
aperture');ylabel ...
    ('\xi_r - Aperture to beam radius ratio a/\omega');
```